Physics for the Life Sciences – Rotation, Torque and Statics Solutions

Introduction:

Dear student,

Thank you for opening this solution manual for the Rotation, Torque and Statics chapter of the Physics for the Life Sciences Question Manual. This resource has been created by members of the Education Team at WebStraw who have previously taken an introductory university physics course.

Purpose:

This resource is meant to supplement the Physics for the Life Sciences Question Manual, by providing solutions to select questions. To access the full question manual, please click here.

Instructions

We recommend first trying to complete the problems in the question manual on your own. If you get stuck, you can use this resource to view the solution provided by one of our Education Team members. Once you are confident you understand how to solve that question, we recommend solving additional related problems in order to successfully master the topic.

Disclaimer

This resource assumes that you have a basic understanding of key concepts related to the Rotation, Torque and Statics unit in physics. If you are looking to improve your understanding of specific physics content, check out the additional resources provided at the end of the question manual.

Note: There may be more than one correct method to solve some of the problems outlined in the question manual. Thus, the solutions provided may not represent the only acceptable solution.

If you have any comments or feedback regarding this resource or the solutions contained in it, please do not hesitate to contact us at team@webstraw.ca

We wish you the best of luck on your learning journey!

- The WebStraw Education Team

D1:

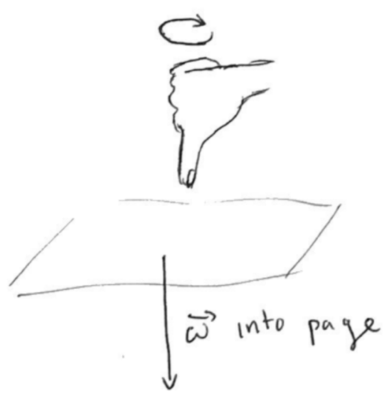
The ball that is sliding will reach the bottom first for 2 main reasons:

1.) The balls start with the same potential energy which can be converted to kinetic energy. However, A's kinetic energy is "divided" between its linear velocity & it's angular velocity while B only has linear kinetic energy $\rightarrow$ this leads to faster linear movement (down the ramp) for B

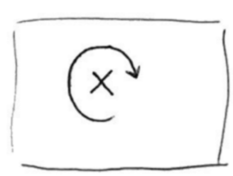
2.) when something is rolling, it doesn't experience much kinetic friction; instead, each segment of Ball A experiences static friction momentarily as it rolls. Static friction has a greater coefficient of friction than kinetic friction, providing a greater "slowing down" force on A.

D3:

Use right hand rule; when thumb points in direction of $\vec{\omega}$, fingers curl in direction of rotation $\rightarrow$ the $\vec{\omega}$ vector is also the axis of rotation



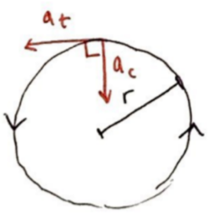
Looking from above page:



The direction of rotation when looking from above the page will be clockwise, around the $\vec{\omega}$ vector

D5:

Yes; tangential acceleration does not affect direction of rotation, only magnitude. Centripetal acceleration changes direction but not velocity.



D9:

Rate of rolling depends on moment of inertia; lower moment of inertia = higher translational (linear) kinetic energy. Assuming the same radius & mass (necessary), the moments of inertia are:

Solid sphere = $\frac{2}{5}MR^2$

Hollow sphere = $\frac{2}{3}MR^2$ $\longrightarrow$ $\frac{2}{5} < \frac{1}{2} < \frac{2}{3}$

Solid cylinder = $\frac{1}{2}MR^2$

Therefore, the solid sphere will reach the bottom first, then the solid cylinder, then the hollow sphere

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D11:

Use right hand rule; a decrease in angular velocity counterclockwise (since the fan will be slowing down) is the same as an increase in angular velocity clockwise. If you curl your fingers clockwise, you will find the angular $\vec{v}$ and $\vec{a}$ vectors point down

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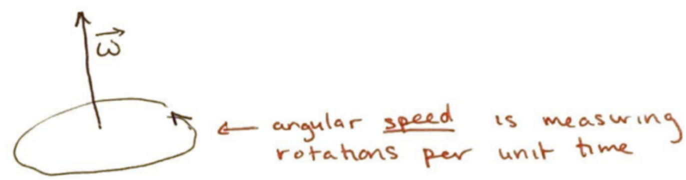
D12:

Remember that angular speed is the rotational speed & occurs in the plane of rotation; do not confuse with angular velocity that is perpendicular to this plane



If angular velocity & acceleration are in opposite directions, there is deceleration happening; similar to if you are moving forward & there is a backwards acceleration. Thus, angular speed will decrease

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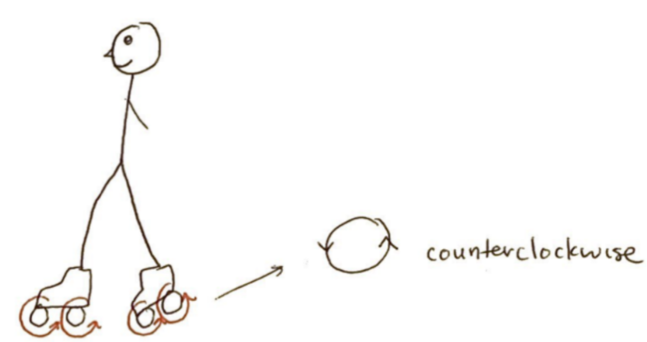
D15:

A bowling ball is a solid sphere, a basketball is a hollow sphere. Since their masses & diameter are the same, the MR^2 term in each moment of inertia can be ignored.

$$\frac{\text{Moment of inertia of bowling ball}}{\text{Moment of inertia of basketball}} = \frac{\frac{2}{5}\cancel{MR^2}}{\frac{2}{3}\cancel{MR^2}} = \frac{2}{5}\Big/\frac{2}{3} = \frac{2}{5} : \frac{2}{3} = 3:5$$

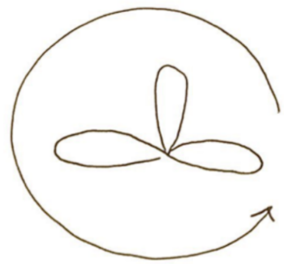
D17a:

When you rollerblade forwards, the rotating appears as shown below:



Using right hand rule, the angular velocity will be directed out of the page, or to the point of view of the rollerblader, to the left.

D17b:



Using right hand rule, if rotation is CCW in the plane of the page, the angular velocity vector is coming out of the page.

19a:

Given: angular speed = 10 rotations per second (0.1s per rotation)

Required: angular velocity

Solve: angular velocity = $\frac{d\theta}{dt}$ $\longrightarrow$ each rotation is 2π radians

$$= \frac{10 \cdot 2\pi}{s}$$

$$= \boxed{20\pi \text{ radians/second}}$$

19b:

Given: $r = 0.2\text{ cm} = 0.002\text{m}$

Required: Centripetal acceleration & minimum coeff. of static friction

Solve: $a_c = \omega^2 r = (20\pi)^2(0.002) = \boxed{7,896 \frac{m}{s^2}}$

Centripetal force $F_c = m a_c = m\omega^2 r$

static force must equal this to prevent slipping so $F_s = F_c$

$$F_s = F_c$$

$$\mu_s \cancel{m} g = \cancel{m}\omega^2 r$$

$$\mu_s = \frac{\omega^2 r}{g} = \frac{7.896 \frac{m}{s^2}}{9.8 \frac{m}{s^2}} = \boxed{0.806}$$

D21:

Given: deceleration $= 2.33 \frac{rad}{s^2}$ $\omega_i = 18 \frac{rad}{s}$ $\omega_f = 0 \frac{rad}{s}$

Required: time for bike to stop

Solve: $\omega_f = \omega_o + at \longrightarrow t = \frac{\omega_f - \omega_o}{a} = \frac{0 \frac{rad}{s} - 18 \frac{rad}{s}}{-2.33 \frac{rad}{s^2}} = \boxed{7.73 s}$

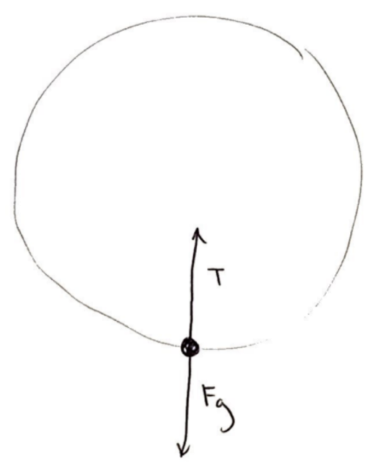
negative

23a:

Given: $m = 100g = 0.1 kg$ $r = 2m$ T at bottom = 20N

Required: speed at bottom of circle

Solve: At bottom of circle, the ball's FBD looks like this:



The <u>net</u> force (our theoretical "centripetal force") = T − mg

$$F_c = T - mg$$

$$m\omega^2 r = 20N - (0.1 kg)(9.8 \frac{m}{s^2})$$

$$m\frac{v^2}{r} = 19.02 N$$

$$v = \sqrt{\frac{(r)(19.02 N)}{m}} = \sqrt{\frac{(2m)(19.02N)}{(0.1 kg)}} = 19.5 \frac{m}{s}$$

D25:

Given: $\omega_{max} = 75 \frac{\text{rotations}}{\text{minute}}$ $\omega_i : 88$ rpm $\omega_f = 84$rpm after 2.5 turns

Required: Rate of angular acceleration in rads/s and time to reach safe speed

Solve: $2.5 \text{ turns} = 2.5 \cdot 360° = 900° = \theta = 5\pi$

$\omega_f^2 = \omega_0^2 + 2a\theta$

$84 \text{ rpm} = 84 \cdot \left(\frac{2\pi \text{ rads}}{\text{rotation}}\right)\left(\frac{1 \text{ minute}}{60 s}\right) = 8{,}796 \text{ rads}/s$

$88 \text{ rpm} = 9{,}215 \text{ rads}/s$

negative for deceleration

$8.796^2 = 9.215^2 + 2a(5\pi) \longrightarrow a = \boxed{-0{,}24 \frac{\text{rads}}{s^2}}$

remember our "real" ω_f now is 75 rpm

$t = \frac{\omega_f - \omega_0}{a}$

$= \frac{75 \text{ rpm} - 8.796 \text{ rads}/s}{-0{,}24 \frac{\text{rads}}{s^2}} = \frac{7{,}854 \frac{\text{rads}}{s} - 8{,}796 \frac{\text{rads}}{s}}{-0{,}24 \frac{\text{rads}}{s^2}}$

$= \boxed{3{,}925 \text{ s}}$

D27:

Given: $r = 0{,}24 m$ $m = 0{,}625 kg$ $\vec{V} = 10 \frac{m}{s}$ $\omega = 5 \frac{rad}{s}$

Required: Kinetic energy of the ball

Solve: The ball has linear kinetic energy & rotational kinetic energy

Linear kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}(0.625 kg)(10 \frac{m}{s})^2$

$= 31{,}25 J$

Rotational kinetic energy $= \frac{1}{2} I \omega^2$ where I = moment of inertia

I of hollow sphere $= \frac{2}{3} mR^2$ so

rotational kinetic energy $= \frac{1}{2}\left(\frac{2}{3} mR^2\right)\omega^2$

$= \frac{1}{2}\left(\frac{2}{3}\right)(0.625)(0.24)^2(5)^2$

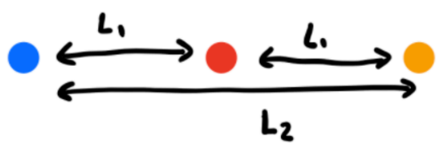
$= 0.3 \text{ J}$

Total kinetic energy = linear E_k + rotational E_k $= 31.25 \text{ J} + 0.3 \text{ J}$

$= \boxed{31.55 \text{ J}}$

No solution for D29 provided

D31)



★ $I_1 = m_1 L_1^2$ ★ $I_2 = m_2 L_2^2 = m_2 (2L_1)^2$

combo

$$\frac{I_1}{I_2} = \frac{m_1 L_1^2}{4 m_2 L_1^2} = \frac{1}{4} \cdot \frac{m_1}{m_2}$$

$\therefore$ if $m_1 = m_2$ then m_2 is 4x greater inertia

D33) $I = \frac{2}{5} mR^2$ $V = R\omega \Rightarrow \omega = \frac{V}{R}$

- R' radius of loop > R radius of disc

a) Conservation of energy

$PE_1 + \cancel{KE_1} = \cancel{PE_1} + KE_{tran} + KE_{rot}$

$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I \omega^2$

$mgh = \frac{1}{2} mv^2 + \frac{1}{2}\left(\frac{2}{5} m\cancel{R^2}\right)\left(\frac{V}{\cancel{R}}\right)^2$

$$mgh = \tfrac{7}{10}mv^2$$

$$v = \sqrt{\tfrac{10}{7}gh} \text{ @ bottom ramp}$$

b)

$$\omega = \frac{v}{R} = \sqrt{\frac{10gh}{7R^2}} \text{ @ bottom ramp}$$

c) Q = ramp height

$E_{\text{initial solid disc}} = mgQ$

disc @ top of loop $(R') \Rightarrow F_{net} = F_N + mg = \frac{mv_{top}^2}{R'}$

for min Q, disc move @ $v_{top} \Rightarrow mg = \frac{mv_{top}^2}{R'} \Rightarrow v = \sqrt{R'g}$

Energy of disc @ top

$$E_{Top} = PE_{top} + KE_{tran,top} + KE_{rot,top} = mg(2R') + \tfrac{1}{2}mv_{top}^2 + \tfrac{1}{2}I\omega_{top}^2$$

$$= mg(2R') + \tfrac{1}{2}mv_{top}^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\left(\frac{v_{top}}{R}\right)^2$$

$$E_{Top} = 2mgR' + \tfrac{7}{10}mR'g$$

$$\cancel{mg}Q = 2\cancel{mg}R' + \tfrac{7}{10}\cancel{m}R'\cancel{g}$$

$$Q = \tfrac{27}{10}R' \text{ min height for disc}$$

d) $E_{initial} = mgQ$, non-rotating block

Energy of block @ top

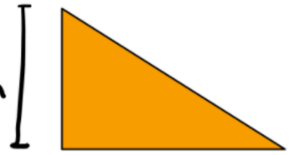
$$E_{Top} = PE_{top} + KE_{tran,top} + \cancel{KE_{rot,top}} = mg(2R') + \tfrac{1}{2}mv_{top}^2$$

$$E_{Top} = mg(2R') + \tfrac{1}{2}m(R'g)^2 = \tfrac{5}{2}mR'g$$

$$\cancel{mg}Q = \tfrac{5}{2}\cancel{m}R'\cancel{g}$$

$$Q = \tfrac{5}{2}R' \text{ min height for block}$$

D35)



m = mass

① full of bean

$PE_i + \cancel{KE_i} = \cancel{PE_f} + KE_f$

$mgh = \frac{1}{2}mv^2 \Rightarrow KE_{full} = mgh$

② empty bean

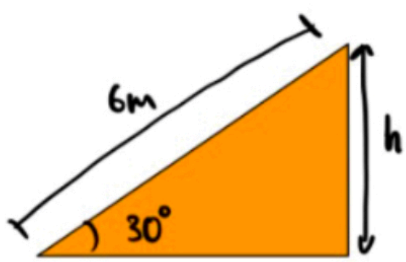
$m = \frac{1}{4}m$

$KE_{empty} = \frac{1}{4}mgh$

Combo $\frac{KE_{full}}{KE_{empty}} = \frac{mgh}{\frac{1}{4}mgh}$

$KE_{full} = 4KE_{empty}$

D37) $\theta = 30°$ $d_{cylinder} = 9.8m$ $m = 4.6kg$



Solid cylinder : $I = \frac{1}{2}mr^2$

$V = r\omega$

$\sin 30 = \frac{h}{6}$

$h = 3m$

Conservation of energy

$PE_i = KE_{tran} + KE_{rot}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\cancel{m}gh = \frac{1}{2}\cancel{m}(r\omega)^2 + \frac{1}{2}(\frac{1}{2}\cancel{m}r^2)\omega^2$

$$gh = \tfrac{3}{4} r^2 \omega^2$$

$$\omega = \sqrt{\frac{4gh}{3r^2}}$$

$\omega = 1.28$ rad/s

$V = r\omega = 6.24$ m/s $\quad\therefore$ cylinder speed 6.24 m/s

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D39)

$F_{net} = mg\sin\theta - F_f$

$ma = mg\sin\theta - F_f$

$ma = mg\sin\theta - \frac{I\alpha}{r^2}$

① $a = \dfrac{g\sin\theta}{1 + \frac{I}{mr^2}}$

Torque balance

$F_f \cdot r = I \cdot \alpha$

$F_f = \frac{I\alpha}{r}$ — $\frac{a}{r}$

$= \frac{Ia}{r^2}$

Assume

$m = 1$ kg

#2 Kin law

$\Delta x = v_i t + \frac{1}{2} a t^2$

Sphere

Time

$v_i = 0 \qquad I = \frac{2}{5} mr^2$

$\Delta x = \frac{7}{\sin 30°} = 14$ m (7 — height)

frm eqn ① $a_s = 3.5$ m/s²

$\Delta x = \cancel{v_i t} + \frac{1}{2} a t^2$

$t_s = \sqrt{\frac{2\Delta x}{a}}$

$t_s = 2.83$ s

$>$

Cylinder

$v_i = 0 \qquad I = \frac{1}{2} mr^2$

$\Delta x = \frac{6}{\sin 30°} = 12$ m (6 — height)

frm eqn ① $a_c = 3.27$ m/s²

$\Delta x = \cancel{v_i t} + \frac{1}{2} a t^2$

$t_c = \sqrt{\frac{2\Delta x}{a}}$

$t_c = 2.71$ s

$\therefore$ $t_s > t_c$ so cylinder is quicker to reach bottom

Velocity

Conservation of energy

$$\cancel{KE_i} + PE_i = KE_f + \cancel{PE_f}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \frac{v}{r}$$

Sphere

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

$$gh = \frac{7}{10}v^2$$

$$v_s = \sqrt{\frac{10gh}{7}} = 9.9 m/s$$

Cylinder

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{1}{2}\cancel{m}\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2$$

$$gh = \frac{3}{4}v^2$$

$$v_c = \sqrt{\frac{4gh}{3}} = 8.9 m/s$$

$\therefore$ $v_s > v_c$ so solid sphere faster @ bottom

D41) $T = 2.2s$ $\quad r = 28cm = 0.28m$ $\quad m = \text{mass}$

$v = \omega r$

a) $\omega = \frac{2\pi}{T} = 2.85 s^{-1}$

$a = \omega^2 r = 2.27 m/s^2$

b) $F = ma = 2.27 \cdot m$ N

c) $v = \mu r g$

$\omega \cancel{r} = \mu \cancel{r} g$

$\mu = \frac{\omega}{g} = 0.29$

D43) $\omega_0 = 380\ \text{rad/min} = 39.8\ \text{rad/s}$ $\qquad t = 10.0\,s$

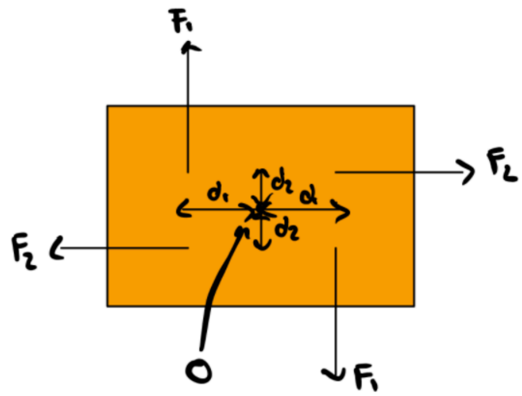
$\omega = 1400\ \text{rad/min} = 146.6\ \text{rad/s}$ $\qquad \tau = 2680\ N/m$

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{\omega - \omega_0}{t} = 10.68\ \text{rad/s}^2$$

$$I = \frac{\tau}{\alpha} = 243.4\ kg \cdot m^2$$

D45)



$\tau_0 = \vec{r}_\perp \times \vec{F}$

assume O is the center of the net torque for all forces

$$\vec{\tau}_0 = d_1 F_1\ cw + d_2 F_2\ cw + d_1 F_1\ cw + d_2 F_2\ cw$$

$$= 2(d_1 F_1 + d_2 F_2)\ cw$$

cw = clockwise

$$F_{net} = F_1 \hat{y} - F_1 \hat{y} + F_2 \hat{x} - F_2 \hat{x}$$

$$F_{net} = 0\ N$$

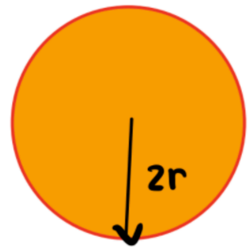
D47) Assume angle btwn F and r 90°

$I = mr^2$

$\tau = I \cdot \alpha$

$F = x$
(α_s)

$\alpha_s = \frac{I}{\tau} = \frac{x}{mr^2}$



$F = 2x$
(α_d)

$\alpha_d = \frac{I}{\tau} = \frac{2x}{m(2r)^2} = \frac{x}{2mr^2}$

$\therefore \alpha_s > \alpha_d$

① $x = \alpha_s mr^2$ ② $x = 2\alpha_d mr^2$

combo

$\alpha_d = \frac{\alpha_s}{2}$

$\therefore$ Force difference and larger wheel require ½ α compare to smaller

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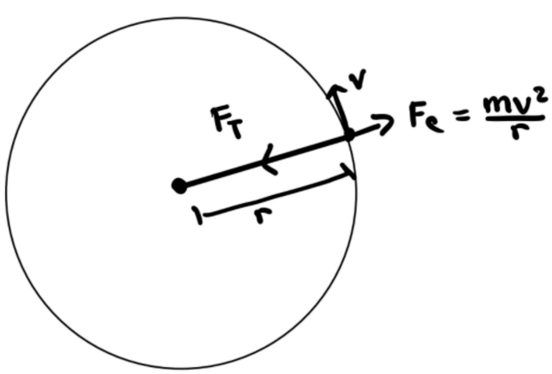
D49)



A $T_A = \frac{mv^2}{r}$

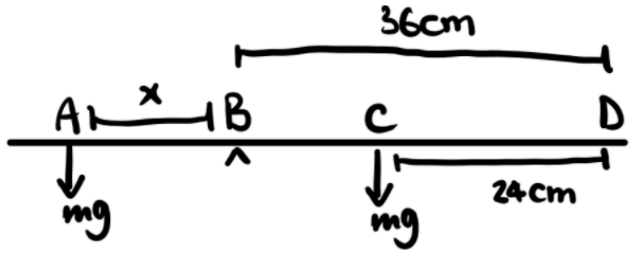
B $T_B = \frac{2mv^2}{r}$

C $T_C = \frac{mv^2}{2r}$

D $T_D = \frac{2mv^2}{2r} = \frac{mv^2}{r}$

$\therefore T_B > T_A = T_D > T_C$

D51)



assume τ_{net} @ $B = 0 \Rightarrow$ cw τ by mg = ccw τ by mg

$\Delta x_{BC} = 12cm = 0.12m$

$\Delta x_x = ?$

$m_{rod} = 0.28kg \qquad m_{sus} = 0.55kg$

$$\tau_{AB} = \tau_{BC}$$

$$m_{sus}\cancel{g}\,\Delta x_x = m_{rod}\cancel{g}\,\Delta x_{BC}$$

$$\Delta x_x = \frac{m_{rod}\,\Delta x_{BC}}{m_{sus}} = 0.061m$$

$\therefore$ distance of 0.55kg frm higher end = $x + BD \Rightarrow 0.42m$

D53)

m = mass per block

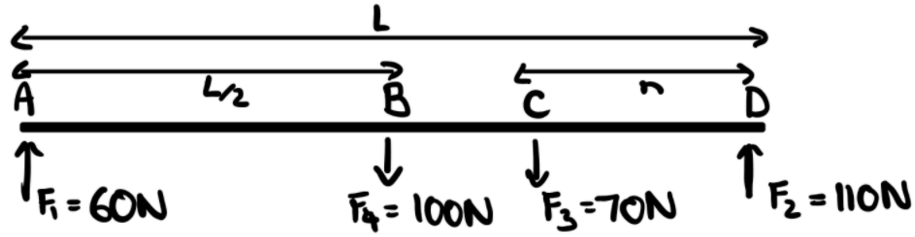
L = length of plank

Center of mass frm right $= \dfrac{\cancel{3m(0)} + 2\cancel{m}(\frac{L}{2}) + 2\cancel{m}(L)}{(3+2+2)\cancel{m}}$

$$= \frac{3L}{7}$$

$\therefore$ center of mass @ 42.9% of length frm right end

D55)



$$F_1 + F_2 = F_3 + F_4$$

Frm right end (D) torque

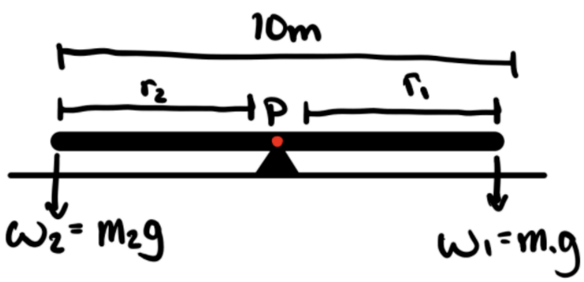
$$F_1 \cdot L + \cancel{F_2 \cdot 0} = F_3 \cdot n + F_4 \cdot L/2$$

$$F_1 \cdot L - F_3 \cdot n - F_4 \cdot L/2 = 0$$

$$n = \frac{1}{7} L$$

$\therefore$ C is 14.3% frm right end

D57)



$m_1 = 70kg$

$r_1 = 3.4m$

$m_2 = 54kg$

$$\tau_{child\,1} = \tau_{child\,2}$$

$$\omega_1 r_1 = \omega_2 r_2$$

$$r_2 = \frac{\omega_1 r_1}{\omega_2} = \frac{m_1 \cancel{g} r_1}{m_2 \cancel{g}}$$

$$r_2 = 4.57m$$

$\therefore$ Child 2 should sit 4.57m frm left side